A Model of Quantum Electrodynamics with Higher Derivatives

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Abstract

A new version of application Pauli-Villars regularized Green functions in the quantum field theory using higher derivatives is proposed. In this version the regularizing mass M is large but finite. Our approach is demonstrated and discussed on the example of QED. It is shown that in our case there are no ultraviolet divergences and - on the example of the selfenergy spinor Feynman diagram - no infrared ones.

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1 Introduction

One of the unwritten rules of the standard quantum field theory is the limited order of the derivatives in the field equations. As is well known this order is two for the field with integer spin and - one for half spin. This rule appears, at first, with the Klein-Gordon and Dirac equations, which stand in the origin of many field models. We can suppose that the analogy with the classical mechanics is one of the main reason for this situation - let us recall that only first order time derivatives enter the action of the usual mechanical systems. However, mechanical actions with higher time derivatives were considered sometimes by different authors [1]. In this case they use the special canonical formalism first developed by Ostrogradski [2]

Recently models with higher derivatives appeared also in quantum field theory, especially in the two dimensional one (see for example [3] where the reader can find other references on this subject). However, we are going to concentrate our attention to the use of higher derivatives in the 3+1 dimensional quantum field theory. In this direction we can point out works in which the authors study several mathematical properties of the free field equations with higher derivatives. These equations are characterized by a polynomial in the d'Alambertian operator [4]. It is interesting to mention the work [5], where the higher derivative kinetic term was introduced in the Nambu-Jona-Lasinio model.

In the last few years appeared several works in which the authors study the so called "differential regularization" first proposed by Freedman, Johnson and Latorre [6]. Though the derivatives are used there only for the regularization of the loop diagram in the coordinate representation, we may suppose that these formal rules maybe follow from the appropriate quantum field theory with higher derivatives.

The main difficulty arising when we use higher derivatives in the quantum field theory is the appearance of the indefinite metric in the state space and all physical consequences from this. That is why quantum field theory with higher derivatives in the free part of the corresponding Lagrangean can not exist, if we want every vector from the state space to have a physical meaning (here we have in mind the spaces of in- and out- states). However, there exists another approach in which the physical states form the appropriate subspace with definite metric in the frame of the whole indefinite state space. Such an approach was sketched by Hawking [7]. In this case the physical theory

appears as immersed in a wider theory, analogously to the situation in the quantum electrodynamics in the Lorentz gauge.

Different authors adduce different arguments for the application of higher derivatives depending on the concrete problems they pay attention to. The presence of higher derivatives in the kinetic part of the field Lagrangean leads to free propagation function in which the power of the momentum p is less than -2. This fact means that the divergences of the Feynman diagrams will become smaller. For example if the propagator of the fermions in the usual spinor QED has the behaviour p^{-3}, when $p \to \infty$, then all well known divergent diagrams become finite. This fact is the main argument for us to consider higher derivatives. The aim of the present paper is to give an alternative formulation of the spinor QED in which the fermion field obeys a third order differential equation.

2 The Model

We are going to consider a model in which the main fields are the spinor field $\varphi(x)$ and vector one $A_\mu(x)$. Besides we have an auxiliary spinor and scalar fields denoted by $\psi(x)$ and $\Phi(x)$ respectively. To write down the action S of our model, let us introduce the following notation

$$D_\mu = \partial_\mu - ieA_\mu \qquad \nabla_\mu = \partial_\mu - ie\partial_\mu\Phi \tag{1}$$

where e is the dimensionless electric charge. (From here on the Greek indices such as (μ, ν, λ) run from 0 to 3 - as vector indices - those such as (α, β, γ) run from 1 to 4 - as spinor indices). Then

$$S = \int [L_{sp}(\phi) + L_v(A) + L^T(\phi)] d^4x \tag{2}$$

where the parts L_{sp} and L_v of the Lagrangean have the following form:

$$\begin{aligned} L_{sp}(\varphi) &= \frac{1}{\sqrt{M^2 - m^2}}(\nabla^\star_\mu \bar{\varphi} \nabla^\mu \psi + \nabla^\star_\mu \bar{\psi} \nabla^\mu \varphi) + \bar{\varphi}(i/2\gamma^\mu \overleftrightarrow{D}_\mu + m)\varphi + \\ &\quad + \bar{\psi}(i/2\gamma^\mu \overleftrightarrow{\nabla}_\mu - m)\psi - \frac{m^2}{\sqrt{M^2 - m^2}}(\bar{\psi}\varphi + \bar{\varphi}\psi) \qquad (3) \\ L_v(A) &= -1/2\partial^\mu(A_\nu - \partial_\nu\Phi)\partial_\mu(A^\nu - \partial^\nu\Phi) \qquad (4) \end{aligned}$$

m and M are mass parameters. Here with $\nabla^{\star}_{\mu}$ and $\bar{\varphi}$ we have denoted the complex and Dirac conjugation respectively. Moreover we have denoted:

$$u\overleftrightarrow{D}_{\mu}v = uD_{\mu}v - D_{\mu}u.v$$

$$u\overleftrightarrow{\nabla}_{\mu}v = u\nabla_{\mu}v - \nabla_{\mu}u.v$$

$L^T(\Phi)$ is the free kinetic part of our Lagrangean for the scalar field Φ. As we can see below the concrete form of this term does not matter for our model.

For the construction of the Lagrangean (3) we have used, in general, a known manipulation for the exclusion of the highest derivative (in our case - third derivative), with the help of the auxiliary field $\psi(x)$. It is easy to verify, that if we replace the field $\psi(x)$ by new a auxiliary field $\chi(x)$ with the relation

$$\psi(x) = \chi(x) - \frac{1}{\sqrt{M^2 - m^2}}(i\gamma^{\mu}\nabla_{\mu} + m)\varphi(x) \tag{5}$$

then we can obtain the following new form of the spinor part of the Lagrangean

$$\begin{aligned} L_{sp}(\varphi) \;\; &= \;\; -\frac{1}{M^2 - m^2}\nabla^{\star}_{\mu}\bar{\varphi}(i/2\gamma^{\mu}\overleftrightarrow{\nabla}_{\mu} + m)\nabla^{\mu}\varphi \\ &+ \;\; \frac{m^2}{M^2 - m^2}\bar{\varphi}(i/2\gamma^{\mu}\overleftrightarrow{\nabla}_{\mu} + m)\varphi \\ &+ \;\; \bar{\varphi}(i/2\gamma^{\mu}\overleftrightarrow{D}_{\mu} + m)\varphi + \bar{\chi}(i/2\gamma^{\mu}\overleftrightarrow{\nabla}_{\mu} - m)\chi \end{aligned} \tag{6}$$

without the change of the action (2). We see that the free field χ is not connected with the rest of the fields and has no significance for our model. In spite of the equivalence between the spinor part (3) of the Lagrangean and $L_{sp}(\varphi)$ from (6) we give our preferences to the one from eq. (3), because it is more suitable for the applications of the canonical quantum formalism.

The Lagrangean parts $L_{sp}(\varphi)$ and $L_v(A)$ are gauge invariant under the following local transformations:

$$\begin{aligned} \varphi(x) &\to \exp ie\eta(x).\varphi(x); \quad \psi(x) \to \exp ie\eta(x).\psi(x) \\ A_{\mu}(x) &\to A_{\mu}(x) + \partial_{\mu}\eta(x); \quad \Phi(x) \to \Phi(x) + \eta(x) \end{aligned} \tag{7}$$

where $\eta(x)$ is the corresponding local parameter.

Let us try then to write down $L^T(\Phi)$ in the gauge invariant form. In this case it has to depend on $\partial_\mu\Phi - A_\mu$, because this is the only gauge invariant combination in the presence of the field Φ:

$$L^T(\Phi) = W(\partial_\mu\Phi - A_\mu) \tag{8}$$

In this case the action (2) becomes gauge invariant. However, with a simple redefinition of the fields ψ, φ and A_μ with the help of the gauge transformations (7), the field Φ will disappear from the action. This means that the free Lagrangean $L^T(\Phi)$ can not be gauge invariant for arbitrary $\eta(x)$. Taking into account that the conformal dimension of $\Phi(x)$ is zero, it is natural to suppose that the free part of its equation must be $\Box^2\Phi$ ($\Box$ is d'Alambertian operator). Then the gauge invariance is of the type (7) but with $\eta(x)$ satisfying the equation

$$\Box^2\eta(x) = 0 \tag{9}$$

As we shall see below this restriction is in accordance with the usually used gauge fixing rules in QED.

Now we can give the explicit form of $L^T(\Phi)$:

$$L^T(\Phi) = \partial^\mu\Phi\partial_\mu U + \frac{1}{2}U^2 \tag{10}$$

$U(x)$ is an auxiliary field with the help of which $L^T(\Phi)$ depends on first derivatives only. The gauge transformation of $U(x)$ has the form

$$U(x) \to U(x) + \Box\eta(x)$$

Then we can obtain the field equations. Varying the action (2) we have the primary equations:

$$\sqrt{M^2 - m^2}(i\gamma^\mu D_\mu + m)\varphi = (\nabla^2 + m^2)\psi \tag{11}$$

$$\sqrt{M^2 - m^2}(i\gamma^\mu \nabla_\mu - m)\psi = (\nabla^2 + m^2)\varphi \tag{12}$$

$$\Box(A_\mu - \partial_\mu\Phi) = j_\mu^{el} \tag{13}$$

$$\Box\partial^\mu(A_\mu - \partial_\mu\Phi) = \Box U - \partial^\mu j_\mu^\Phi \tag{14}$$

$$\Box\Phi = U \tag{15}$$

where we have used the following notation:

$$j_\mu^{el} = -e\bar{\varphi}\gamma_\mu\varphi \tag{16}$$

$$\begin{aligned} j_\mu^\Phi &= -\frac{ie}{\sqrt{M^2-m^2}}(\bar{\varphi}\nabla_\mu\varphi + \bar{\psi}\nabla_\mu\varphi - \nabla_\mu^\star\bar{\varphi}.\psi - \nabla_\mu^\star\bar{\psi}.\varphi) - \\ &- e\bar{\psi}\gamma_\mu\psi + \partial^\nu w_{\mu\nu} \end{aligned} \tag{17}$$

Here $w_{\mu\nu}$ is an arbitrary antisymmetric tensor and the last term in the right-hand side of eq. (17) expresses the arbitrariness of the dependence of the current j_μ^Φ from its divergence (note that only $\partial^\mu j_\mu^\Phi$ enter the field equations - eq. (14)).

As we mentioned above there are some auxiliary fields in our model. It is necessary to exclude them and obtain the final form of the field equations. After simple calculations we have from eqs. (11) and (12) :

$$(\nabla^2 + m^2)(i\gamma^\mu\nabla_\mu + m)\varphi + (M^2 - m^2)(i\gamma^\mu D_\mu + m)\varphi = 0 \tag{18}$$

and from eqs. (14), (15) -

$$\Box\partial^\mu(A_\mu - \partial_\mu\Phi) = \Box^2\Phi - \partial^\mu j_\mu^\Phi \tag{19}$$

Furthermore from eqs. (13), (19) we can obtain

$$\Box^2\Phi = \partial^\mu(j_\mu^{el} + j_\mu^\Phi) \tag{20}$$

It is easy to verify that the right hand side term in the last equation is zero, i.e.,

$$\partial^\mu(j_\mu^{el} + j_\mu^\Phi) = 0 \tag{21}$$

as a result from field equations (11) and (12). The calculations leading to this result one can see in the Appendix. Then

$$\Box^2\Phi = 0 \tag{22}$$

and instead of eq. (19) we have

$$\Box\partial^\mu(A_\mu - \partial_\mu\Phi) = -\partial^\mu j_\mu^\Phi \tag{23}$$

Now we can point out the final equations. These are the equations (18), (13) and (22) for the basic fields and one of the eqs. (11) or (12) for the auxiliary field ψ (we ignore the field U). Certainly we must consider our current j_μ^Φ with excluded ψ. The rest of the equations (21) and (23) follow from the basic ones.

To complete the description of our model, let us introduce new basic fields using the above mentioned partial gauge invariance. This we make with the help of the following expressions:

$$\varphi(x) = \exp ie\Phi(x).\phi(x); \qquad A_\mu(x) = \mathcal{A}_\mu(x) + \partial_\mu\Phi \tag{24}$$

Then the operator ∇_μ turns into ∂_μ in all equations and instead of (18), (13) and (23) we have:

$$\frac{1}{M^2 - m^2}(\Box + M^2)(i\gamma^\mu\partial_\mu + m)\phi + e\gamma^\mu\mathcal{A}_\mu\phi = 0 \tag{25}$$

$$\Box\mathcal{A}_\mu = j_\mu^{el} \tag{26}$$

$$\Box\partial^\mu\mathcal{A}_\mu = \partial^\mu j_\mu^{el} \tag{27}$$

respectively. As we can see we have excluded the current (17) from eq. (23) using the identity (21). Then equation (27) becomes a direct consequence of eq. (26).

3 Quantization of the model

We are going to quantize the model described above with the help of the perturbation theory in Dirac (interaction) representation [3] For this matter we will apply the approach proposed by Bogolubov [8]. As is well known, this approach involves the formulation of the quantization of the corresponding free field theory. In our case we have from eq.(3) the following free spinor Lagrangean:

$$\begin{aligned} L^0(\phi) &= \frac{1}{\sqrt{M^2 - m^2}}(\partial_\mu\bar\phi\partial^\mu\psi + \partial_\mu\bar\psi\partial^\mu\phi) + \bar\phi(i/2\gamma^\mu\overleftrightarrow{\partial}_\mu + m)\phi + \\ &+ \bar\psi(i/2\gamma^\mu\overleftrightarrow{\partial}_\mu - m)\psi - \frac{m^2}{\sqrt{M^2 - m^2}}(\bar\psi\phi + \bar\phi\psi) \end{aligned} \tag{28}$$

[3] Sometimes this representation is called Tomonaga-Shwinger representation.

Remark. Let us remind that when we are passing to the quantum theory all products of the fields in the Lalgrangeans must be understood as normal ones. This convention makes unnecessary the use of any additional symbols for the normal product.

The Lagrangean for the field $\mathcal{A}_\mu$ is the well known Lagrangean for the massless vector field and its quantization as electromagnetic field in the Lorentz gauge is given, e.g., in [8]. The field Φ satisfying the eq. (22) played an important role in the conformal invariant QED and its quantum theory is well known too (see for example the works [9]). Moreover, we have seen that this field can be excluded from our model with the help of the gauge transformations (24). On the other hand let us remind that the field $\psi(x)$ is an auxiliary spinor field which has to be excluded too. However, the Lagrangean (28) is more suitable for canonical quantization (containing first derivatives only) than one from (6). That is why here we will consider the theory with the Lagrangean (28) only. The free field equations in our case have the form:

$$\frac{1}{\sqrt{M^2 - m^2}}(i\gamma^\mu \partial_\mu + m)\phi = (\Box + m^2)\psi \tag{29}$$

$$\frac{1}{\sqrt{M^2 - m^2}}(i\gamma^\mu \partial_\mu - m)\psi = (\Box + m^2)\phi \tag{30}$$

The corresponding canonical momenta are

$$\pi_\psi(x) = \frac{1}{\sqrt{M^2 - m^2}}\partial_0\bar{\phi}(x) + \frac{i}{2}\psi^\star(x) = (\pi_{\bar{\psi}}(x))^\star\gamma^0 \tag{31}$$

$$\pi_\phi(x) = \frac{1}{\sqrt{M^2 - m^2}}\partial_0\bar{\psi}(x) + \frac{i}{2}\phi^\star(x) = (\pi_{\bar{\phi}}(x))^\star\gamma^0 \tag{32}$$

The anticommutator of the field $\phi(x)$ we denote as

$$\Gamma_{\alpha\beta}(x) = \{\phi_\alpha(x), \bar{\phi}_\beta(0)\} \tag{33}$$

($\{...\}$ means anticommutator). From equations (29) and (30) we have the following equation for the function $\Gamma_{\alpha\beta}(x)$:

$$(\Box + M^2)(i\gamma^\mu \partial_\mu + m)\Gamma(x) = 0 \tag{34}$$

Furthermore using the canonical commutation relations:

$$\{\pi_\phi(x), \phi(y)\}_{x_0=y_0} = \{\pi_\psi(x), \psi(y)\}_{x_0=y_0} = i\delta^3(x)\delta_{\alpha\beta} \tag{35}$$

we have the following initial conditions for the function Γ

$$\begin{aligned} \Gamma_{\alpha\beta}(x)|_{x_0=0} = \partial_0\Gamma_{\alpha\beta}(x)|_{x_0=0} = 0 \\ \partial_0^2\Gamma_{\alpha\beta}(x)|_{x_0=0} = (M^2 - m^2)\gamma^0_{\alpha\beta}\delta^3(x) \end{aligned} \tag{36}$$

To obtain the latter we have used the anticommutation relations between the fields ψ and ϕ, and the equations they satisfy. These calculations are very simple and we omit them here.

Now it is easy to obtain our function Γ:

$$\Gamma_{\alpha\beta}(x) = -i(i\gamma^\mu\partial_\mu - m)_{\alpha\beta}[D_m(x) - D_M(x)] \tag{37}$$

where D_m and D_M are the scalar Pauli-Jordan functions with masses m and M respectively. Analogously one can obtain the rest of the singular functions from which we will write down here the causal Green function only:

$$\Gamma^c_{\alpha\beta}(x) = -i(i\gamma^\mu\partial_\mu - m)_{\alpha\beta}[D^c_m(x) - D^c_M(x)] \tag{38}$$

D^c_m and D^c_M are corresponding scalar causal Green functions for the masses m and M:

$$D^c_m(x) = \frac{1}{(2\pi)^4}\int \frac{e^{-ikx}}{m^2 - k^2 - i\varepsilon} d^4k$$

Our causal Green function (38) is normalised in such a way that it coincides with the propagation function:

$$< T\phi_\alpha(x), \bar{\phi}_\beta(0) >_0 = \Gamma^c_{\alpha\beta}(x) \tag{39}$$

The formulae (37) and (38) show us that the free quantum theory of our field ϕ is containing indefinite metric in state space. This fact becomes more clear if we consider the concrete form of the general solution for the field ϕ . It has the form

$$
\begin{aligned}
\phi_\alpha(x) &= \phi^0_\alpha(x) + \\
&+ \frac{1}{(2\pi)^{3/2}}\sum_{r=1,2}\int\{[\sqrt{\frac{\Omega(M+m)}{2M}}a^{+r}(\mathbf{p})v^{+r}_\alpha(\Omega,\mathbf{p}) + \\
&+ \sqrt{\frac{\Omega(M-m)}{2M}}c^{-r}(\mathbf{p})v^{-r}_\alpha(\Omega,\mathbf{p})]e^{ipx} + \\
&+ [\sqrt{\frac{\Omega(M-m)}{2M}}b^{+r}(\mathbf{p})v^{+r}_\alpha(\Omega,\mathbf{p}) + \\
&+ \sqrt{\frac{\Omega(M+m)}{2M}}d^{-r}(\mathbf{p})v^{-r}_\alpha(\Omega,\mathbf{p})]e^{-ipx}\}\frac{d^3p}{\Omega} \qquad (40)\\
&\Omega = \sqrt{\mathbf{p}^2+M^2}\\
&\mathbf{p} = (p_1,p_2,p_3) \qquad \mathbf{p}^2 = \sqrt{p_1^2+p_2^2+p_3^2}
\end{aligned}
$$

Here $\phi^0_\alpha(x)$ is the usual free spinor field satisfying Dirac equation

$$(i\gamma^\mu\partial_\mu + m)\phi^0(x) = 0$$

and the following anticommutation relation

$$\{\phi^0_\alpha(x), \bar{\phi}^0_\beta(y)\} = -i(i\gamma^\mu\partial_\mu - m)_{\alpha\beta}D_m(x-y) \qquad (41)$$

The mode-vectors $v^{\pm\alpha}(\Omega,\mathbf{p})$ fulfill the usual spinor identities which in our case have the form

$$(\gamma^\mu p_\mu \mp M)v^{\pm itr}(\Omega,\mathbf{p}) = 0 \qquad (42)$$

Moreover this mode-vectors satisfy the normalization condition which can be written down in the following two forms:

$$\sum_{\alpha=1}^{4}(v^{\pm r}_\alpha(\Omega,\mathbf{p}))^\star v^{\pm s}_\alpha(\Omega,\mathbf{p}) = \delta_{rs} \qquad (43)$$

$$or$$

$$\sum_{\alpha=1}^{4}(\overline{v^{\pm r}_\alpha(\Omega,\mathbf{p})})v^{\pm s}_\alpha(\Omega,\mathbf{p}) = \pm\frac{M}{\Omega}\delta_{rs} \qquad (44)$$

The relation

$$\sum_{\alpha=1}^{4}(v_{\alpha}^{\pm r}(\Omega,\pm\mathbf{p}))^{\star}v_{\alpha}^{\mp s}(\Omega,\mp\mathbf{p})=0 \tag{45}$$

is a condition for the orthogonality of the mode-vectors and finally the relation

$$\sum_{r=1,2}v^{\pm r}(\Omega,\mathbf{p})(\overline{v^{\pm r}(\Omega,\mathbf{p})})=\frac{\gamma^{\mu}p_{\mu}\pm M}{2\Omega} \tag{46}$$

expresses the completeness of these vectors.

The quantities $a^{+r}(\mathbf{p})$, $b^{+r}(\mathbf{p})$, $c^{-r}(\mathbf{p})$ and $d^{-r}(\mathbf{p})$ entering the general solution (40) are operators in the quantum theory and they must satisfy the following anticommutation relations:

$$\begin{aligned}\{a^{+r}(\mathbf{p}),a^{-s}(\mathbf{q})\} &= -\Omega\delta_{rs}\delta^{3}(\mathbf{p}-\mathbf{q})\\ \{b^{+r}(\mathbf{p}),b^{-s}(\mathbf{q})\} &= -\Omega\delta_{rs}\delta^{3}(\mathbf{p}-\mathbf{q})\\ \{c^{+r}(\mathbf{p}),c^{-s}(\mathbf{q})\} &= -\Omega\delta_{rs}\delta^{3}(\mathbf{p}-\mathbf{q})\\ \{d^{+r}(\mathbf{p}),d^{-s}(\mathbf{q})\} &= -\Omega\delta_{rs}\delta^{3}(\mathbf{p}-\mathbf{q})\end{aligned} \tag{47}$$

(all other anticommutators are zero) according to the (33). In the above written formulas we have made the following notation

$$\begin{aligned}a^{-r}(\mathbf{p})&=(a^{+r}(\mathbf{p}))^{\star}\\ b^{-r}(\mathbf{p})&=(b^{+r}(\mathbf{p}))^{\star}\\ c^{+r}(\mathbf{p})&=(c^{-r}(\mathbf{p}))^{\star}\\ d^{+r}(\mathbf{p})&=(d^{-r}(\mathbf{p}))^{\star}\end{aligned} \tag{48}$$

The new quantities $(a^{+r}(\mathbf{p}))^{\star},(b^{+r}(\mathbf{p}))^{\star},(c^{-r}(\mathbf{p}))^{\star},(d^{-r}(\mathbf{p}))^{\star}$ appear in the field $\bar{\phi}_{\alpha}(x)$ Dirac conjugated to the field (40) and the sign $\star$ has the meaning of Hermitean conjugation.

Now let us describe the structure of our state space S. First of all S is containing the subspace H which coincides with the Fock space of the free quantum field $\phi^{0}(x)$. This is the usual state space of the free quantum spinor field with positive metric. Its vacuum state $\mid 0>$ is common to the whole space S and satisfies the following additional conditions:

$$\begin{aligned}a^{-r}(\mathbf{p})\mid 0> &= b^{-r}(\mathbf{p})\mid 0>=c^{-r}(\mathbf{p})\mid 0>=d^{-r}(\mathbf{p})\mid 0>=0\\ <0\mid a^{+r}(\mathbf{p}) &= <0\mid b^{+r}(\mathbf{p})=<0\mid c^{+r}(\mathbf{p})=<0\mid d^{+r}(\mathbf{p})=0\end{aligned} \tag{49}$$

Then we can see that S contains an additional subspace G in which the basis is formed from all monomials of creation operators $(a^{+r}(\mathbf{p}), b^{+r}(\mathbf{p}), c^{+r}(\mathbf{p})$, $d^{+r}(\mathbf{p}))$ acting on the vacuum. The corresponding operators with the sign "-" are the annihilation operators. According to the our commutation relations given above, we can do the following conclusions:

i) The metric of the space G given with the usual scalar product of the Fock space is indefinite.

ii) The spaces H and G are orthogonal to each other and

$$S = H \otimes G$$

It is obvious that the vectors of the space S can not be used as physical states because of the presence of the ghosts from subspace G.

However, we can define the physical space to coincide with the subspace H and consider only its vectors as edge physical states. Then it is easy to verify that

$$< ph' \mid \bar{\phi}(x)\gamma^{\mu}\phi(x) \mid ph >=< ph' \mid \bar{\phi}^{0}(x)\gamma^{\mu}\phi^{0}(x) \mid ph >$$

according to the eq.(40) and of course

$$\partial_{\mu} < ph' \mid \bar{\phi}(x)\gamma^{\mu}\phi(x) \mid ph >= 0 \tag{50}$$

where $\mid ph >$ and $\mid ph' >$ are two arbitrary states from the physical space H. Moreover, in this case the relation (50) define the space H and can be considered as a defining condition for the physical states.

Passing to our model with interaction it is naturally to generalize the condition (50) for the definition of the physical states in the interaction case. This we can do defining the latter with the help of the relation, formally coinciding with relation (50) but with $\phi(x)$ satisfying the equations (25)-(27). Such definition is in accordance, except for the free case, with the quantum theory in the interaction representation, where the spinor physical space coincides with the space H.

As we mentioned above we consider our model only on the physical space. This means that all matrix elements of operators and their products having some physical meaning must by taken between physical states only. Then the nonphysical states such as ghost ones will give contribution to the intermediate virtual states only.

Now we are going to see what happens with our model in the physical space. According to the condition (50) we can obtain there the form of the equations (26) and (27):

$$< ph' \mid \Box \mathcal{A}_\mu(x) - j^{el}_\mu(x) \mid ph >= 0 \tag{51}$$

and

$$< ph' \mid \Box \partial^\mu \mathcal{A}_\mu(x) \mid ph >= 0 \tag{52}$$

The last equation shows us that in the chosen physical space automatically appears the gauge condition such as

$$\Box \partial^\mu \mathcal{A}_\mu = 0$$

This condition contains the Lorentz gauge fixing

$$\partial^\mu \mathcal{A}_\mu = 0$$

which means that the our physical space has a subspace in which equation (51) coincides with the Maxwell equation in the Lorentz gauge. The vectors belonging to this restricted physical space are denoted here as $\mid ph_0 >$. Then instead of eqs. (51) and (52) we have

$$\begin{aligned} &< ph'_0 \mid \Box \mathcal{A}_\mu(x) - j^{el}_\mu(x) \mid ph_0 >\equiv \\ &\equiv< ph'_0 \mid \partial^\nu F_{\nu\mu}(x) - j^{el}_\mu(x) \mid ph_0 >= 0 \\ &F_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu \end{aligned} \tag{53}$$

and

$$\partial^\mu < ph'_0 \mid \mathcal{A}_\mu(x) \mid ph_0 >= 0 \tag{54}$$

The construction described here obtains its concrete form in the interaction representation, where, as is well known, the Lagrangean and the other physical quantities are expressed through the corresponding free quantum fields in the Heisenberg representation. In this case the mentioned above restricted physical space H_0 in which equations (53) and (54) take place, coincides with the following space with nonnegative metric:

$$H_0 = H \otimes R_a \tag{55}$$

where the space R_a can be defined as

$$R_a \equiv \bigoplus_{N\geq 0} \bigotimes_{k=0}^{N} \mathbf{l}_k^1 \tag{56}$$
$$\mathbf{l}_0^1 =\mid l_0^1 >\equiv\mid 0 >$$

and where $\mathbf{l}_k^1$ are the one particle spaces for the field $\mathcal{A}_\mu(x)$. The arbitrary one particle state in this case has the form

$$\mid l^1 >= \int \mathcal{A}_\mu^+(x) l^\mu(x) d^4x \mid 0 > \tag{57}$$

where $\mathcal{A}_\mu^+(x)$ is the positive frequency part of the field $\mathcal{A}_\mu(x)$ and $l^\mu(x)$ are arbitrary test functions fulfilling the condition:

$$\partial_\mu l^\mu(x) = 0 \tag{58}$$

i.e.,

$$\partial^\mu \mathcal{A}_\mu^-(x) \mid l' >= \frac{1}{i} \int \partial^\mu D_0^-(x-y) l_\mu(y) d^4x \equiv 0$$

Here $\mathcal{A}_\mu^-(x)$ and $D_0^-(x-y)$ are the negative frequency parts of the field $\mathcal{A}_\mu(x)$ and the massless Pauli Jordan function respectively.

Our physical space is the subspace of the whole quantum state space Q which in the considered representation has the structure

$$Q = S \otimes F_A; \qquad H_0 \subset Q \tag{59}$$

where F_A is the Fock space of the quantum vector field $\mathcal{A}_\mu(x)$ satisfying the d'Alambert equation.

Now we can formulate the main result of the present paper:

The quantum model with the action (2) and Lagrangeans (3), (4) and (10), leading to the field equations with the higher derivatives (25), (26) and (27), contains in its own state space Q the subspace H_0 with nonnegative metric in which our model coincides with the spinor QED. From this point of view the latter is immersed in our model described above. It is necessary to mention that this QED is in regularized form, because the spinor propagation function is given by expression (38) and, as is easy to see, it coincides with the Pauli-Villars regularized one. That is why the Feynman diagrams are free from divergences in our case. In the next section we are going to discuss these questions.

4 Regularized QED

In this section we would like to compare the described above regularized QED (RQED) which is immersed in our model, with the standard one. This comparison we are going to do on the level of Feynman diagrams and it helps to understand better our reasons to consider such theory.

As we have seen, the physical space of RQED in the interaction representation is composed from Fock spaces of the free spinor $\phi^0_\alpha(x)$ and free photon $A_\mu(x)$ fields in Lorentz gauge. This space is with nonnegative metric and gives us the possibility to do the next steps to obtain the physical space of the free electrons, positrons and photons, e.g., passage to the Coulomb gauge fixing, factorization of the zero norm state vectors and so on (see ref. [9]); the steps usually done in standard QED too. Then it is easy to notice, that our physical space is the same as in standard QED. There is no difference also between the photon propagation functions in the two theories. In our case this function is

$$\Delta^c_{\mu\nu}(x) = \eta_{\mu\nu} D^c_0(x)$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor and $D^c_0(x)$ is the causal Green function of the d'Alambert equation.

Remark. In our model we have used the Lorentz gauge fixing as the most simple. However, there are no obstacles to use other forms of the photon part of our RQED. For instance if we had used, instead of Lagrangean (4), a new one as follows:

$$L_v(A) = -1/2\partial^\mu(A_\nu - \partial_\nu\Phi)\partial_\mu(A^\nu - \partial^\nu\Phi) + \frac{\kappa}{2}\partial^\mu(A_\nu - \partial_\nu\Phi)\partial^\nu(A_\mu - \partial_\mu\Phi)$$

the corresponding equations for the photons on the physical states would have obtained the form

$$\Box\mathcal{A}_\nu - \kappa\partial_\nu\partial^\mu\mathcal{A}_\mu = j^{el}_\nu$$

with the following gauge fixing condition

$$\Box\partial^\mu\mathcal{A}_\mu = 0$$

This form of the equation for the electromagnetic field $\mathcal{A}_\mu$ is well known too (see ref. [10])

The main difference between RQED and the standard QED is in the form of the spinor propagation function. As it is seen from eq. (38) the form of our spinor propagation function coincides with the well known Dirac one but regularized with the help of the Pauli-Villars regularization without performing the limit $M \to \infty$. In the momentum representation the function (38) has the behaviour p^{-3} for $p \to \infty$, so RQED is free of ultraviolet divergences. However, a new parameter M appears here. For the understanding of the influence of M over the RQED we are going to consider all Feynman diagrams arranged in two groups. In the first group we put all diagrams which remain finite in the limit $M \to \infty$. These diagrams correspond to converging ones in the standard QED. Here it is possible to choose M large enough so that the corresponding diagrams from the two QED's will coincide with each other on such a range of the values of the external momenta, which could have any experimental significance. That is why we can say that the first group of diagrams RQED differs from standard QED only for very big external momenta, the range of values of which is defined by the value of the mass parameter M.

In the second group of Feynman diagrams we put those which correspond to the divergent diagrams of QED. These diagrams increase unlimitedly for large M. In the standard QED over these diagrams one applies the renormalization procedure. In the our case, because of the finiteness there is no similar necessity in the RQED. That is why all nonuniqueness appearing in QED after the infinite renormalization is described here through the parameter M only. In our opinion this is the main difference between the two considered here QED's. We would like to demonstrate this difference with the help of some example. For this we choose the second order electron-positron self-energy Feynman diagram, i.e., one of the basic divergent diagrams in standard QED.

As usual the mentioned diagram we denote as $\Sigma_{\alpha\beta}(p)$, where p is the external electron-positron momentum. Without giving here the calculation which is well known, we will start from the following expression for this quantity in our case:

$$\Sigma_{\alpha\beta}(p) = \frac{e^2}{8\pi^2}\int_0^1 d\xi(2m - \gamma^\mu p_\mu \xi)_{\alpha\beta} ln\frac{\xi p^2 - M^2 + i\epsilon}{\xi p^2 - m^2 + i\epsilon} \tag{60}$$

The integration in the right hand side of the last equation can be taken and for us is interesting the result when $M^2 \gg p^2$, i.e., in the low energy range.

Then we have

$$\Sigma(p) = \frac{e^2}{16\pi^2}(4m - \gamma^\mu p_\mu) ln\frac{M^2}{m^2} + \Sigma'(p) \tag{61}$$

where we have denoted by $\Sigma'(p)$ the part which does not increase with M

$$\Sigma'(p) = \frac{e^2}{8\pi^2}\int_0^1 d\xi(2m - \gamma^\mu p_\mu) ln\frac{m^2}{m^2 - \xi p^2} + O(\frac{1}{M}) \tag{62}$$

The corresponding diagram in the standard QED after renormalization has the form

$$\Sigma(p) = c_1(\gamma^\mu p_\mu - m) + c_2 + \Sigma'(p) \tag{63}$$

where $\Sigma'(p)$ is the same as in eq. (62) (see ref. [8]). Here c_1 and c_2 are arbitrary finite constants in the result of the renormalization. Comparing eq. (63) with eq. (61) we can see that in our case both constants depend on M as follows

$$c_1 = \frac{-e^2}{16\pi^2} ln\frac{M^2}{m^2} \qquad c_2 = \frac{3e^2 m}{16\pi^2} ln\frac{M^2}{m^2}$$

This means that in RQED we have no nonuniqueness appearing in the process of the calculation of such a diagram.

Remark: It is possible to do finite renormalization of our model. Then the analogous nonuniqueness will appear in RQED too. However, the finiteness of the considered diagrams makes such procedure unnecessary.

The only quantities which can be considered as arbitrary parameters are the mass parameters M and m. Recalling that $\Sigma_{\alpha\beta}(p)$ is the second order correction to the free spinor operator we can write down the corresponding corrected Green function in the momentum representation:

$$G(p) = \frac{1}{\frac{1}{M^2 - m^2}(p^2 - M^2)(\gamma^\mu p_\mu - m) - \Sigma(p)} \tag{64}$$

Now we can calculate the pole-point of this function which will represent the electron-positron physical mass m_{ph}. Up to second order, this mass can be obtained in the form:

$$\begin{aligned} m_{ph} &= m + \delta m \\ \delta m &= -\frac{3e^2 m}{16\pi^2} ln\frac{M^2}{m^2} - \frac{5e^2 m}{32\pi^2} \end{aligned} \tag{65}$$

It is well known that the point $p^2 = m^2$ is a branching point of $\Sigma'(p)$ and because of the absence of the renormalization it is impossible to set $m_{ph} = m$. That is why there are bare m and dressed m_{ph} electron-positron masses in RQED, connected by expression (65). The dressed mass m_{ph} is a pole of the Green function (64) and it is less than the bare mass m which is a branching-point of the same Green function. Here these points are automatically different and there is no necessity in RQED to introduce small photon mass to reach this difference, as it takes place in the standard QED. Perhaps this is the most significant property. If we reformulate it, this property means that there are no infrared divergences in the considered diagram (see for this ref.[8]).

5 Conclusion

We defined a model with higher derivatives. Our analysis shows that in the suitable chosen physical subspace of states, this model coincides with QED. The only difference between QED and our model (calling here RQED) is the usage of the regularized causal spinor Green function in the latter. However, in our theory we can not take $M \to \infty$, because the mass parameter M enter the initial spinor part of the Lagrangean and it has a finite value. After simple analysis we saw that RQED is free from ultraviolet divergences. Choosing the parameter M sufficiently large, we can say that RQED and QED have the same behaviour in the low energy range. We are hopeful that RQED will have a behaviour in the high energy range better than QED, because of the faster decrease of our spinor propagator for large momentum.

On the example of the spinor selfenergy second order Feynman diagram we saw that there is no infrared divergence too.

From the obtained results we can see that there exists an alternative way to consider given particle theory in which the field operator describes not only the physical one. Then the physical theory turns out as immersed into wider theory, such that the latter has contribution to the intermediate (virtual) states only. We have demonstrated here this approach on the well known theory for comparison. However, there are theories such as nonrenormalizable ones, where this approach could turn out to be the only possible. For example, applying our propagator to the four-fermions interaction, the corresponding theory will have no divergences at all.

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APPENDIX

First of all let us write down the full current $j_\mu^{el} + j_\mu^{\Phi}$ in the terms of fields defined in eq. (24). Then we have

$$j_\mu^{el} + j_\mu^{\Phi} = -e\partial^\mu \bar{\phi}\gamma_\mu\phi - e\partial^\mu\bar{\xi}\gamma_\mu\xi -$$

$$-\frac{ie}{\sqrt{M^2 - m^2}}(\bar{\phi}\partial_\mu\xi + \bar{\xi}\partial_\mu\phi - \partial_\mu\bar{\phi}.\xi - \partial_\mu\bar{\xi}.\phi)$$

where by $\xi(x)$ we have denoted

$$\xi(x) = \exp[-ie\Phi(x)].\psi(x)$$

Then the equations (11) and (12) can be written down as follows:

$$\sqrt{M^2 - m^2}(i\gamma^\mu\partial_\mu + m)\phi = (\Box + m^2)\xi - e\mathcal{A}_\mu\gamma^\mu\phi \tag{A.1}$$

$$\sqrt{M^2 - m^2}(i\gamma^\mu\partial_\mu - m)\xi = (\Box + m^2)\phi \tag{A.2}$$

We can do the verification of eq. (21) with direct calculations. For this let us write down the Dirac conjugates to eqs. (A.1) and (A.2)

$$\sqrt{M^2 - m^2}(-i\partial_\mu\bar{\phi}\gamma^\mu + m\bar{\phi}) = (\Box + m^2)\bar{\xi} - e\bar{\phi}\gamma^\mu\mathcal{A}_\mu \tag{A.3}$$

$$\sqrt{M^2 - m^2}(-i\partial_\mu\bar{\xi}\gamma^\mu - m\bar{\xi}) = (\Box + m^2)\bar{\phi} \tag{A.4}$$

Now we must multiply eqs. (A.1) and (A.3) by $\bar{\phi}$ from left and $-\phi$ from right respectively and then add the results. Analogously we must proceed with eqs. (A.2) and (A.4). It is easy to see then, that the sum of the two obtained in this manner identities coincides with the expression:

$$\partial^\mu(j_\mu^{el} + j_\mu^{\Phi}) = 0$$

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